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Unit-Cube Expression for Space-Charge Resistance

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A simple analysis shows that the unit-cube conductance is a figure of merit in semiconductor device design theory. The unit-cube conductance, G , is given by $2Kv_d$ where K is the permittivity of the semiconductor and v_d is the limiting drift velocity.

The space-charge resistance, R_{sc} , due to carrier generated under avalanche condition is derived for p-n junctions. It is found that for parallel-plane structure, $R_{sc} = 1/GN$, where N is the number of unit cubes in the depletion region with cube edge equal to the depletion width or $N = A/W^2$ where W is the depletion width and A the junction area. The disturbance in voltage caused by the space-charge effect is given by $I/GN = JW^2/G$ where I and J are the current and current density, respectively. Similar results are obtained for p-n junctions with coaxial-cylinder and concentric-sphere structures.

For silicon, the value of G is approximately $40 \mu\text{mhos}$. The transconductance of a silicon surface-controlled avalanche transistor in terms of the unit-cube expression is about $12.5 N \mu\text{mhos}$.

A simple analysis of "avalanche resistance" can be given for the limiting case in which carriers are generated at one boundary surface of the depletion region of a p-n junction and travel across the depletion region with a limiting drift velocity v_d . Structures satisfying these conditions can be of the n^+pp^+ form. It will be shown that the quantity

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$2Kv_d$ (where K is the permittivity of the semiconductor) is a figure of merit in semiconductor device design theory which limits the performance of space-charge-limited devices. This quantity is a combination of "material constants" similar to $F_B v_d$ (where F_B is "breakdown field") which limits the frequency-power performance of transistors^{1,2} and K/σ (σ is the conductivity) which limits the gain-bandwidth product³ of solid-state devices.

For a structure in which the space-charge layer is bounded by parallel planes of area A and spacing W , it will be shown that the effective space-charge resistance can be interpreted as due to N unit-cube conductances in parallel, where the unit-cube conductance G is given by

$$G = 2Kv_d \quad (1)$$

and N is number of unit cubes in the depletion layer with cube edge equal to the depletion width, or

$$N = A/W^2. \quad (2)$$

The space-charge resistance is then given by

$$R_{sc} = 1/NG. \quad (3)$$

For coaxial-cylinder and concentric-sphere structures, similar results are obtained for the R_{sc} . The number of unit cubes (or curvilinear cubes), however, depends on the radius of the surface upon which avalanche occurs and the length of the cylinder (for the coaxial-cylinder structure). These functional dependences are derived below.

An interesting application of the space-charge resistance and the unit-cube expression is given for a surface-controlled avalanche transistor (SCAT).⁴

I. PARALLEL PLANE STRUCTURE

As represented in Fig. 1(a) the depletion layer of an n^+pp^+ structure extends through the p layer with a doping of N_a , and is bounded by the planes at $x = 0$ and $x = W$. When the applied voltage V is equal to the breakdown voltage V_B the electric field $E(x)$ has its maximum absolute value F_B at $x = 0$ and decreases to $F_B - (qN_a W/K)$ at $x = W$. This insures breakdown at $x = 0$. Furthermore, if $qN_a W/K < 0.9 F_B$, then the field is everywhere $\geq F_B/10$, so that holes have their limiting drift velocity v_d all across W .

The space-charge current, I , is given by

$$I = v_d \rho A, \quad (4)$$

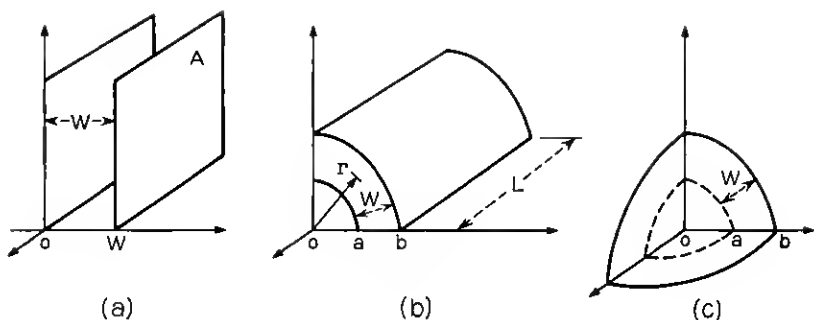


Fig. 1—p-n junction geometry of (a) parallel plane, (b) coaxial cylinder, and (c) concentric sphere structures.

where ρ is the carrier-charge density and A the area. Since E at $x = 0$ is assumed to be equal to F_B , the disturbance $\Delta E(x)$ in the electric field due to ρ is

$$\Delta E(x) = \frac{Ix}{AKv_d} \quad (5)$$

so that the disturbance in voltage caused by the carriers (i.e., the average field times W) is obtained by integrating $\Delta E(x)$

$$\Delta V_B = I \left(\frac{W^2}{A} \right) \left(\frac{1}{2Kv_d} \right) = \frac{I}{NG}. \quad (6)$$

The total voltage is thus

$$V = V_B + \Delta V_B = \left(F_B W - \frac{qN_a W^2}{2K} \right) + \frac{I}{NG} = V_B + IR_{sc}, \quad (7)$$

which verifies the interpretation of G and N .

II. COAXIAL CYLINDER STRUCTURE

Consider first that the maximum field occurs at the inner surface. As shown in Fig. 1(b) the depletion layer extends through the intrinsic region of an n^+ip^+ coaxial-cylinder structure and is bounded by the cylinders of radii $r = a$ and $r = b$. When $V = V_B$, the electric field $E(r)$ has its maximum absolute value F_B at $r = a$, and decreases to $F_B a/b$ at $r = b$.

The space-charge current per unit length, I/L is given by

$$\frac{I}{L} = 2\pi r \rho v_d \quad (8)$$

so that ρ varies as $1/r$. Integrating Poisson's equation leads to a disturbance $\Delta E(r)$ in the electric field and ΔV_B in the voltage due to ρ given by

$$\Delta E(r) = \frac{\rho(r-a)}{K} = \frac{I}{2\pi K v_d L} \left(1 - \frac{a}{r}\right) \quad (9)$$

and

$$\Delta V_B = \frac{I}{(2K v_d)\pi L} \left[b - a - a \ln \left(\frac{b}{a}\right) \right] \equiv \frac{I}{NG} = IR_{sc}, \quad (10)$$

where

$$\frac{1}{N} = \frac{b}{\pi L} \left(1 - \frac{a}{b} - \frac{a}{b} \ln \frac{b}{a}\right) = \frac{\left[2b^2 \left(1 - \frac{a}{b} - \frac{a}{b} \ln \frac{b}{a}\right)\right]}{2\pi b L} \equiv \frac{A_c}{2\pi b L}. \quad (11)$$

A_c is the area on the outer cylinder surface that corresponds to one unit-cube conductance G .

$$\begin{aligned} A_c &\rightarrow (b-a)^2, & \text{for } a \rightarrow b \\ A_c &\rightarrow 2b^2, & \text{for } a \rightarrow 0. \end{aligned} \quad (12)$$

Equation (10) may be interpreted as the resistance of N unit curvilinear cubes in parallel. These cubes are formed by intersection of equipotential surfaces with the orthogonal family of electric field lines. Each cube has a conductance $(2K v_d)$, and the number of cubes N is given by (11). The area A_c approaches $(b-a)^2$ when $a \rightarrow b$, and approaches $2(b-a)^2$ when $a \rightarrow 0$, and consequently remains finite even as the inner cylinder approaches a line.

The maximum field may be caused to occur on the outer surface $r = b$ by adjusting the chemical charges in the depletion layer appropriately, such as a p^+pn^+ structure with the pn^+ junction at $r = b$, the p^+p boundary at $r = a$. In this case, the area A_c approaches $(b-a)^2$ when $a \rightarrow b$, and approaches $2b^2 \ln(b/a)$ when $a \rightarrow 0$. Hence, the space-charge resistance has the same value as given by (10) and (11) when $a \rightarrow b$, but approaches infinity as $a \rightarrow 0$.

III. CONCENTRIC SPHERE STRUCTURE

As shown in Fig. 1(c), the depletion layer extends through the intrinsic region of an n^+ip^+ concentric sphere structure and is bounded by the spheres of radii $r = a$ and $r = b$. When $V = V_B$, the electric field $E(r)$ has its maximum value F_B at the inner surface $r = a$, and decreases to $F_B a^2/b^2$ at $r = b$.

The space-charge current is given by

$$I = 4\pi r^2 \rho v_d . \quad (13)$$

The quantities $\Delta E(r)$ and ΔV_B are given as follows:

$$\Delta E(r) = \frac{\rho(r-a)}{K\epsilon^2} = \frac{I}{4\pi K v_d r} \left(1 - \frac{a}{r}\right) \quad (14)$$

$$\Delta V_B = \frac{I}{(2K v_d) 2\pi} \left(\ln \frac{b}{a} - 1 + \frac{a}{b}\right) \equiv \frac{I}{NG} = IR_{sc} , \quad (15)$$

where

$$\frac{1}{N} = \frac{\left(\ln \frac{b}{a} - 1 + \frac{a}{b}\right)}{2\pi} \equiv \frac{A_s}{4\pi b^2} \quad (16)$$

$$A_s = 2b^2 \left(\ln \frac{b}{a} - 1 + \frac{a}{b}\right)$$

$$A_s \rightarrow (b-a)^2, \quad \text{for } a \rightarrow b \quad (17)$$

$$A_s \rightarrow \infty, \quad \text{for } a \rightarrow 0$$

and A_s is quantity of area on the outer sphere surface that corresponds to one curvilinear unit-cube conductance G . When a is finite or $a \rightarrow b$, (15) may be interpreted as the resistance of N unit curvilinear cubes in parallel, where each cube has a conductance $(2K v_d)$ and the number of cubes N is given by (16). Unlike the n^+ip^+ coaxial-cylinder case, R_{sc} of the concentric-sphere structure approaches infinite resistance as the inner sphere approaches a point.

F_B can occur on the outer sphere for a p^+pn^+ structure, for example. The results of A_s are the same for both limiting cases as given in (17).

An interesting application of the unit-cube expression is that for a surface-controlled avalanche transistor (SCAT)⁴ with a total junction perimeter of P and a space-charge layer W . Because the space-charge resistance is finite for an n^+ip^+ coaxial cylinder structure as the inner cylinder approaches a line [see (12)], it is feasible to make calculations for SCAT on the basis of an avalanche line source. There are $N = P/W$ such unit cubes around the edge, and the total transconductance, g_m , is⁴

$$g_m = \left(\frac{2}{\pi}\right) K v_d \left(\frac{P}{W}\right) , \quad (18)$$

TABLE I—UNIT-CUBE EXPRESSIONS ($W \equiv b - a$)

Structures	Parallel plane	Coaxial cylinders		Concentric spheres
N (No. of unit cubes)	$\frac{A}{W^2}$	$a \rightarrow b$	$a \rightarrow 0$	$a \rightarrow b$
		$\frac{2\pi Lb}{W^2}$	$\frac{\pi L}{b}$	$\frac{4\pi b^2}{W^2}$
Cube edge (cm)	W	W	$\sqrt{2}b$	W
R_{sc} (ohms)		$\frac{1}{GN} = \frac{1}{(2Kv_d)N}$		

or

$$g_m = \left(\frac{1}{\pi}\right)(2Kv_d N) \equiv \frac{1}{\pi R_{sc}} \quad (19)$$

For a silicon SCAT with a device geometry of $P = 1000 \mu$ and $W = 0.5 \mu$, there are 2000 unit cubes with cube edge 0.5μ . The space-charge resistance is 12.5 ohms, and the transconductance is 25,400 μ mhos.

Another application is to calculate the voltage disturbance ΔV_B in a Read diode.⁶ For a silicon Read diode with drift region of 10μ m and an operating current density of 1000 amp/cm², the value of ΔV_B , as obtained from (6), is approximately 25 volts.

A summary of the number of unit cubes and other pertinent quantities is presented in Table I. It has been shown that the unit-cube conductance ($2Kv_d$) is a figure of merit in semiconductor device design theory. The unit-cube expressions are shown to be useful for calculation of the space-charge resistance.

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